The Direct and Indirect Detection of Weakly Interacting Dark Matter Particles[*]

F. Halzen

University of Wisconsin, Department of Physics, Madison WI 53706
and
University of Hawaii, Department of Physics, Honolulu HI 96822

ABSTRACT

An ever-increasing body of evidence suggests that weakly interacting massive particles (WIMPs) constitute the bulk of the matter in the Universe. We illustrate how experimental data, dimensional analysis and Standard Model particle physics are sufficient to evaluate and compare the potential of detectors searching for such particles either directly (e.g. by their scattering in germanium detectors), or indirectly (e.g. by observing their annihilation into neutrinos in underground detectors).

1. Introduction and Results

It has become widely accepted that most of our Universe is made of cold dark matter particles. Big bang cosmology implies that these particles have interactions of order the weak scale, i.e. they are WIMPs.[1] In the early Universe WIMPs are in equilibrium with photons. When the Universe cools to temperatures well below the mass m_χ of the WIMP their density is Boltzmann-suppressed as $\exp(-m_\chi/T)$ and would, today, be exponentially small if it were not for the expansion of the Universe. At some point, as a result of this expansion, WIMPs drop out of equilibrium with other particles and a relic abundance persists. The mechanism is analogous to nucleosynthesis where the density of helium and other elements is determined by competition between the rate of nuclear reactions and the expansion of the Universe.

For WIMPs to make up a large fraction of the Universe today, i.e. a large fraction of Ω, their annihilation cross section has to be "just right". The annihilation cross section can be dimensionally written as α^2/m_χ^2, where α is the fine-structure constant. It then follows that

$$\Omega \propto 1/\sigma \propto m_\chi^2 . \tag{1}$$

The critical point is that for $\Omega \simeq 1$ we find that $m_\chi \simeq m_W$, the mass of the weak intermediate boson. There is a deep connection between critical cosmological density and the weak scale. Weakly interacting particles which constitute the bulk of the mass of the Universe remain to be discovered. When our galaxy was formed the cold dark matter inevitably clustered with the luminous matter to form a sizeable fraction of the

$$\rho_\chi = 0.4 \text{ GeV/cm}^3 \tag{2}$$

galactic matter density implied by observed rotation curves. Unlike the baryons, the dissipationless WIMPs fill the galactic halo which is believed to be an isothermal

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sphere of WIMPs with average velocity

$$v_\chi = 300 \text{ km/sec} \, . \tag{3}$$

In summary, we know everything about these particles (except whether they really exist!). We know that their mass is of order of the weak boson mass with:

$$\text{tens of GeV} < m_\chi < \text{several TeV} \, . \tag{4}$$

Lower masses are excluded by accelerator and (in)direct searches while masses beyond several TeV are excluded by cosmological considerations. We know that WIMPs interact weakly. We also know their density and average velocity in our Galaxy given the assumption that they constitute the dominant component of the density of our galactic halo as measured by rotation curves.

Two general techniques, referred to as direct (D) and indirect (ID), are pursued to demonstrate the existence of WIMPs.[1] In direct detectors one observes the energy deposited when WIMPs elastically scatter off nuclei. The indirect method infers the existence of WIMPs from observation of their annihilation products. WIMPs will annihilate into neutrinos; massive WIMPs will annihilate into high-energy neutrinos which can be detected in high-energy neutrino telescopes. Throughout this paper we will assume that such neutrinos are detected in a generic Cherenkov detector which measures the direction and, to some extent, the energy of a secondary muon produced by a neutrino of WIMP origin in or near the instrument.[2] It can also detect the showers initiated by electron-neutrinos.

The indirect detection is greatly facilitated by the fact that the sun represents a dense and nearby source of accumulated cold dark matter particles.[3] Galactic WIMPs, scattering off nuclei in the sun, lose energy. They may fall below escape velocity and be gravitationally trapped. Trapped WIMPs eventually come to equilibrium temperature and accumulate near the center of the sun. While the WIMP density builds up, their annihilation rate into lighter particles increases until equilibrium is achieved where the annihilation rate equals half of the capture rate. The sun has thus become a reservoir of WIMPs which we expect to annihilate mostly into heavy quarks and, for the heavier WIMPs, into weak bosons. The leptonic decays of the heavy quark and weak boson annihilation products turn the sun into a source of high-energy neutrinos with energies in the GeV to TeV range.

The performance of future detectors is determined by the rate of elastic scattering of WIMPs in a low-background, germanium detector and, for the indirect method, by the flux of solar neutrinos of WIMP origin. Both are a function of WIMP mass and of their elastic cross section on nucleons. In standard cosmology WIMP capture and annihilation interactions are weak, and we will suggest that, given this constraint, dimensional analysis is sufficient to compute the scattering rates in germanium detectors as well as the neutrino flux from the measured WIMP density in our galactic halo. We will derive and compare rates for direct and indirect detection of weakly interacting particles with mass $m_\chi \simeq m_W$ assuming

1. that WIMPs represent the major fraction of the measured halo density. Their flux is

$$\phi_\chi = n_\chi v_\chi = \frac{0.4}{m_\chi} \frac{\text{GeV}}{\text{cm}^3} \, 3 \times 10^6 \frac{\text{cm}}{\text{s}} = \frac{1.2 \times 10^7}{m_{\chi\,\text{GeV}}} \, \text{cm}^{-2}\text{s}^{-1} \, , \tag{5}$$

where $m_{\chi\,\text{GeV}} \equiv (m_\chi / 1 \text{ GeV})$ is in GeV units.

2. a WIMP-nucleon interaction cross section based on dimensional analysis

$$\sigma(\chi N) = \left(G_F m_N^2 \right)^2 \frac{1}{m_W^2} \equiv \sigma_{\text{DA}} = 6 \times 10^{-42} \, \text{cm}^2 \, , \tag{6}$$

3. that WIMPs annihilate 10% of the time in neutrinos (this is just the leptonic branching ratio of the final state particles in the dominant annihilation channels $\chi\bar{\chi} \rightarrow W^+ W^-$ or $Q\bar{Q}$, where Q is a heavy quark).

Clearly the cross section for the interaction of WIMPs with matter is uncertain. Arguments can be invoked to raise or decrease it. Important points are that i) our choice represents a typical intermediate value, ii) all our results for event rates scale linearly in the cross section and can be easily reinterpreted, and iii) the comparison of direct and indirect event rates is independent of the choice.

We present a simple and totally transparent analysis in which the event rates of detectors are derived from the above assumptions. It finesses all detailed dynamics and gives answers that are sufficiently accurate considering that the mass of the particle has not been pinned down. We will find that the event rate in a direct detector is proportional to the WIMP cross section and flux and the density of targets m_N^{-1}, i.e.

$$\frac{dN_{\text{D}}}{dM} = \frac{1}{m_N} \phi_\chi \sigma_{\text{DA}} N(A_D) = \frac{1.4}{m_{\chi\,\text{GeV}}} \, (\text{kg})^{-1} \, (\text{year})^{-1}, \tag{7}$$

where $\frac{dN_{\text{D}}}{dM}$ represents the number of direct events per unit of target mass. $N(A_D)$ represents the coherent enhancement factor for a nuclear target of atomic number A_D, e.g. 76 for Germanium,

$$N(A) \equiv A^3 \left[\frac{1 + \frac{m_\chi}{m_N}}{A + \frac{m_\chi}{m_N}} \right]^2 \, . \tag{8}$$

The rates for indirect detection are

$$dN_{\text{ID}}/dA \simeq \left\{ 1.8 \times 10^{-2} m_{\chi\,\text{GeV}} \right\} \left\{ \rho(A_{ID}) N(A_{ID}) \right\} \left\{ 1 + 1.9 \times 10^{-4} m_{\chi\,\text{GeV}} \right\}^{-7} \, , \tag{9}$$

where dN_{ID}/dA, in units of $(10^4 \, \text{m}^2)^{-1}(\text{year})^{-1}$, represents the number of events from the sun per unit area A detected by a neutrino telescope. The factor $\{\rho N\}$ should be summed over all elements in the sun. Because of additional nuclear form factor effects which are neglected in Eq. 9 it is adequate to consider oxygen with a solar abundance of $\rho = 1.1$ % and $A_{ID} = 16$ as a "typical" element. The observed average muon energy should be in the range $1/4 \sim 1/6 m_{\chi\,\text{GeV}}$.

The above parametrizations readily lead to the conclusion that the direct method is superior if the WIMP interacts coherently on nuclei (which has been assumed for Eqs. 7–9) and, if its mass is lower or comparable to the weak boson mass m_W. We will show that in all other cases, i.e. for relatively heavy WIMPs and for all WIMPs interacting incoherently, the indirect method is competitive or superior, but it is, of course, held hostage to the successful deployment of high energy neutrino telescopes with effective area in the $\sim 10^4$–10^6 m^2 range and with appropriately low threshold. Especially for heavier WIMPs the indirect technique is powerful because underground high energy neutrino detectors have been optimized to be sensitive in the energy region where the neutrino interaction cross section and the range of the muon are large. A kilometer-size detector probes WIMP masses up to the TeV-range, beyond which they are excluded by cosmological considerations.

For high energy neutrinos the muon and neutrino are aligned, with good angular resolution, along a direction pointing back to the sun. The number of background events of atmospheric neutrino origin in the pixel containing the signal will be small. The angular spread of secondary muons from neutrinos coming from the direction of the sun is well described by the relation[2] $\sim 1.2^\circ\big/\sqrt{E_\mu(\text{TeV})}$. Measurement of muon energy, which may be only up to order of magnitude accuracy in some experiments, can be used to infer the WIMP mass from the angular spread of the signal. The spread contains information on the neutrino energy and, therefore, the WIMP mass. More realistically, measurement of the muon energy can be used to reduce the search window around the sun, resulting in a reduced background.

Before proceeding, we comment on our ansatz for the elastic WIMP-nucleon scattering cross section. The simplest dimensional analysis implies that the cross section is $G_F^2 m_N^2$. This correctly describes the Z-exchange diagram of Fig 1a, which is of the form

$$\sigma \ \sim \ G_F^2 \frac{m_N^2 m_\chi^2}{(m_N + m_\chi)^2} \ . \tag{10}$$
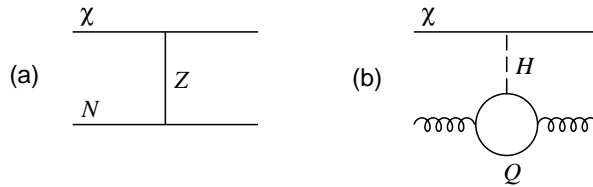

Fig. 1. Examples of (a) incoherent and (b) coherent WIMP-nucleon interactions. In (b) the gluon is a constituent of the target nucleon and Q is a heavy quark.

For coherent interactions, which we will emphasize throughout this paper, there is an additional suppression factor associated with the exchange of the Higgs particle with a mass of order of the weak boson mass; see Fig 1b. In the specific diagram shown the Higgs interacts with the heavy quarks in the gluon condensate associated with the nucleon target. It is of the form

$$\sigma \sim G_F g_H^2 \frac{m_N^2 m_\chi^2}{(m_N + m_\chi)^2} \frac{1}{m_W^2} \ , \tag{11}$$

where $g_H \sim \sqrt{G_F}\, m_N$ describes the condensate. Conservatively, we will use the suppressed WIMP interaction cross section which is appropriate for coherent scattering.

2. Derivation of Detection rates

For the case of direct detection the structure of Eq. 7 is transparent.[1] For indirect detection the number of solar neutrinos of WIMP origin can be calculated in 5 easy steps by determining:

- the capture cross section in the sun, which is given by the product of the number of target nucleons in the sun and the elastic scattering cross section

$$\sigma_\odot = f \left[1.2 \times 10^{57} \right] \sigma_{\mathrm{DA}} . \tag{12}$$

This includes a focussing factor f given, as usual, by the ratio of kinetic and potential energy of the WIMP near the sun. It enhances the capture rate by a factor 10.

- the WIMP flux from the sun which is given by

$$\phi_\odot = \phi_\chi \sigma_\odot / 4\pi d^2 , \tag{13}$$

where $d = 1$ a.u. $= 1.5 \times 10^{13}$ cm.

- the actual neutrino flux, which is obtained after inclusion of the branching ratio. From (5),(6) and (12),(13)

$$\phi_\nu = 10^{-1} \times \phi_\odot = \frac{3 \times 10^{-5}}{m_{\chi\,\mathrm{GeV}}} \,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} . \tag{14}$$

- the probability to detect the neutrino,[2] which is proportional to

$$P = \rho \sigma_\nu R_\mu, \text{ with}$$
$$\rho = \text{Avagadro} \# = 6 \times 10^{23}$$
$$\sigma_\nu = \text{neutrino interaction cross section} = 0.5 \times 10^{-38}\, E_\nu(\mathrm{GeV})\,\mathrm{cm}^2$$
$$R_\mu = \text{muon range} = 500\text{ cm }E_\mu(\mathrm{GeV})$$
$$\text{or} \quad P = 2 \times 10^{-13}\, m_{\chi\,\mathrm{GeV}}^2 \tag{15}$$

Here we assumed the kinematics of the decay chain

$$\chi\bar\chi \;\rightarrow\; W^+ W^-$$
$$\hookrightarrow \mu\nu_\mu$$

with $E_\nu = \frac{1}{2} m_\chi$ (this would be $\frac{1}{3} m_\chi$ for Q decay) and $E_\mu = \frac{1}{2} E_\nu = \frac{m_\chi}{4}$.

- finally, $dN_{\mathrm{ID}}/dA = \phi_\nu P = 1.8 \times 10^{-6}\, m_{\chi\,\mathrm{GeV}}\;(\mathrm{year})^{-1}\,(\mathrm{m}^2)^{-1}$ (16)
  where dN_{ID}/dA represents the number of events from the sun per unit area (m^2) detected by a neutrino telescope.

We can now summarize our results so far by comparing a $10^4 \, \text{m}^2$ neutrino detector, an area typical of the instruments now being deployed, with a kilogram of hydrogen:

$$
\begin{aligned}
dN_{\text{ID}}/dA &= 1.8 \times 10^{-2} m_{\chi \, \text{GeV}} \, (10^4 \, \text{m}^2)^{-1} (\text{year})^{-1} \\
dN_{\text{D}}/dM &= \frac{1.4}{m_{\chi \, \text{GeV}}} \, (\text{kg})^{-1} \, (\text{year})^{-1} \\
\frac{dN_{\text{D}}/dM}{dN_{\text{ID}}/dA} \left(\frac{10^4 \, \text{m}^2}{\text{kg}} \right) &= \frac{7.8 \times 10^1}{m_{\chi \, \text{GeV}}^2}
\end{aligned}
\tag{17}
$$

Direct detection is superior only in the mass range $m_\chi < 10$ GeV, but this region is, arguably, ruled out by previous searches. Indirect detection is the preferred technique. This straightforward conclusion may, however, be invalidated when WIMPs interact coherently with nuclei and targets other than hydrogen are considered. We discuss this next.

3. Coherent Nuclear Enhancements

For WIMPs interacting coherently with nuclei in the detector or in the sun, the nuclear dependence of the event rates resides in

- the target density factor m_N^{-1} in Eq. 7 which is replaced by $(Am_N)^{-1}$,
- the coherent enhancement factor "A^2",
- the nuclear dependence of the cross sections of Eqs. 10,11 which is obtained by the substitution

incoherent

$$
\sigma \sim G_F^2 \frac{m_N^2 m_\chi^2}{(m_N + m_\chi)^2} \to G_F^2 \frac{(Am_N)^2 m_\chi^2}{(Am_N + m_\chi)^2}
$$

coherent

$$
\sigma \sim G_F^2 g_H^2 \frac{m_N^2 m_\chi^2}{(m_N + m_\chi)^2} \frac{1}{m_W^2} \to G_F \left(\frac{g_H}{m_W} \right)^2 \frac{(Am_N)^2 m_\chi^2}{(Am_N + m_\chi)^2} A^2
$$

The coherent enhancement factor for a nucleus A, including a factor A^{-1} for the target density, is therefore given by

$$
\frac{1}{A} \frac{A^2 (Am_N)^2 m_\chi^2}{(Am_N + m_\chi)^2} \frac{(m_N + m_\chi)^2}{m_N^2 m_\chi^2} = A^3 \frac{(m_N + m_\chi)^2}{(Am_N + m_\chi)^2} = A^3 \left[\frac{1 + \frac{m_\chi}{m_N}}{A + \frac{m_\chi}{m_N}} \right]^2 . \tag{18}
$$

This yields Eq. 8.

4. Event Rates for WIMPs with Coherent Interactions

Our simple evaluations, made so far, overestimate the indirect rates for very heavy WIMPS because high energy neutrinos, created by annihilation near the core, may

be absorbed in the sun. Absorption is stronger for neutrinos and, therefore, mostly antineutrinos form the signature for very heavy WIMPS. The probability that an antineutrino escapes without absorption is well parametrized by $(1+3.8 \times 10^{-4} E_\nu)^{-7}$, where $E_\nu \simeq m_\chi/2$. This yields our final result of Eq. 9.

The relative merits of the two methods are illustrated in the following table, which is obtained from Eqs. 7–9 and establishes that a kilogram of germanium and a 10^4 m^2 are competitive.

Table 1. Event rates and signal to background (N/B).

m_χ (GeV)	Direct (/kg/year)		Indirect (/10^4 m^2/year)	
	events	N/B	events	N/B
50	2.2×10^3	7	2.3×10^1	$\simeq 1$
500	1.1×10^3	7	2×10^2	$\simeq 10^2$
2000	2.9×10^2	1	1.7×10^2	$\simeq 10^4$

At the lower energy the event rates for the indirect method are underestimated because also the Earth is a source of neutrinos of WIMP origin.

We conclude that the direct method yields more events for the lower masses, even when compared to a 10^6 m^2 detector. As expected, the indirect method is competitive for heavier WIMPs with a detection rate growing like E_ν^2 or m_χ^2. A 10^5 m^2 instrument covers the full WIMP mass range, even if the WIMPs do not coherently interact with nuclei in the sun. These conclusions are reinforced after considering the signal-to-noise for both techniques which we discuss next.

5. Backgrounds

Indirect Background. For the indirect detection the background event rate is determined by the flux of atmospheric neutrinos in the detector coming from a pixel around the sun.[2] The number of events in a 10^4 m^2 detector is $\sim 10^2/E_\mu(\text{TeV})$ and the pixel size is determined by the angle between muon and neutrino $\sim 1.2°\big/\sqrt{E_\mu(\text{TeV})}$. Using the kinematics $E_\mu \simeq m_\chi/4$ we obtain

$$B_{\text{ID}} = \frac{10^2/E_\mu(\text{TeV})}{2\pi\Big/\left[\frac{1.2° \frac{\pi}{180°}}{\sqrt{E_\mu(\text{TeV})}}\right]^2} = \frac{1.1 \times 10^5}{m_{\chi\,\text{GeV}}^2} \text{ per } 10^4\,\text{m}^2 \text{ per year}$$

This is only valid for large m_χ, i.e. for $E_\mu \cong m_\chi/4 > 100$ GeV. Without this approximation we obtain

E_μ(GeV)	# bkgd. events in 10^4 m^2 in 2π	# pixels of solar size in 2π	bkgd. events per 10^4 m^2 per pixel, per year
10	3200	140	23
100	1060	1.4×10^3	0.8
1000	110	1.4×10^4	8×10^{-3}

For large m_χ the signal to background ratio is

$$\left(\frac{N}{B}\right)_{\text{ID}} \equiv \frac{dN_{\text{ID}}/dA}{dB_{\text{ID}}/dA} \simeq 7.2 \times 10^{-6} m_{\chi\,\text{GeV}}^3$$

Clearly, the extremely optimistic predictions for signal-to-noise are unlikely to survive the realities of experimental physics. One expects, typically, to measure muon energy only to order-of-magnitude accuracy in the initial experiments. The energy of showers initiated by electron neutrinos should be determined to a factor 2. It is not excluded that future, dedicated experiments may do better. The conclusion that high energy muons pointing at the sun represents a superb signature, is unlikely to be invalidated. Direct Background: about 300 events per year per kg.[4] Signal-to-noise therefore exceeds unity up to 2 TeV WIMP mass.

These considerations were used to estimate the signal-to-noise N/B in Table 1.

6. Dynamics?

We emphasize that our results are representative for the specific and much studied example where the lightest supersymmetric particle is Nature's WIMP.[3] Clearly dynamics, which is now defined, can alter our conclusions, but only in "conspiratorial" ways. Dynamics can, on the other hand, increase rates as well, sometimes by well over an order of magnitude, over and above the rates obtained from dimensional analysis in this paper. Our qualitative conclusions are valid, at least in some average sense, in supersymmetry. Our results do, in fact, closely trace the supersymmetry prediction of reference 2 for the choice of Higgs coupling $\frac{g_H^2}{4\pi} = 1$, in their notation.

We feel that the development of detectors should be guided by an analysis like ours rather than by dynamics of theories beyond the standard model for which there is, at present, no experimental guidance.

Acknowledgements

References

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